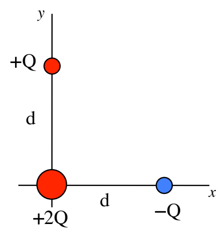
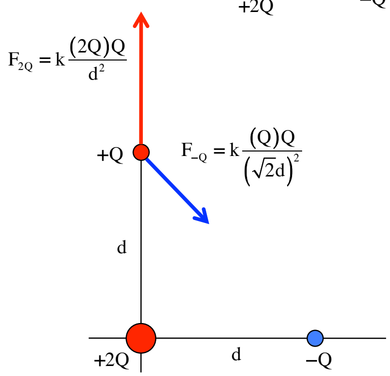
Problem 23.17

Because forces are vectors, we need to determine the coulomb forces on the charge in question AS VECTORS, then break those vectors into their component parts, then sum those components. The only added thrill is relating angles to the geometry. In this case, as the –Q charge will produce a force that is at a 45° angle, we *could* forego that move. But because it is educational, I won't. The force magnitudes are shown to the right with directions (as usual) defined by arrows. The components with angles included are shown on the next page. The summation is also shown there.





Breaking the off-axis force into components requires an angle. Ignoring the fact that the angle is really 45°, we can define it generally as θ (see sketch) and, looking at the physical triangle that exists in the sketch, note that:

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{d}{\sqrt{2}d}$$

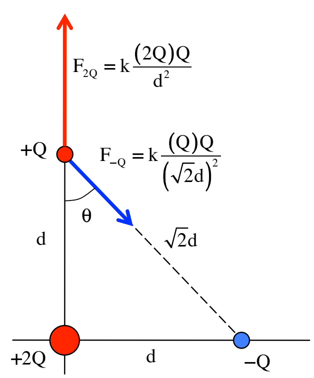
and

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{d}{\sqrt{2}d}$$

Noting additionally that

$$\frac{1}{\sqrt{2}} = .707,$$

we can write the summation of forces in the "x" and "y" directions as:



$$\vec{F} = \left(F_{-Q}\sin\theta\right)\hat{i} + \left(F_{2Q} - F_{-Q}\cos\theta\right)\hat{j}$$

$$= \left(k\frac{(Q)Q}{(\sqrt{2}d)^2}\sin\theta\right)\hat{i} + \left(k\frac{(2Q)Q}{d^2} - k\frac{(Q)Q}{(\sqrt{2}d)^2}\cos\theta\right)\hat{j}$$

$$= \left(k\frac{(Q)Q}{2d^2}\left(\frac{d}{\sqrt{2}d}\right)\right)\hat{i} + \left(k\frac{(2Q)Q}{d^2} - k\frac{(Q)Q}{2d^2}\left(\frac{d}{\sqrt{2}d}\right)\right)\hat{j}$$

$$= k\frac{Q^2}{d^2}\left[\left(\frac{.707}{2}\right)\hat{i} + \left(2 - \frac{.707}{2}\right)\hat{j}\right]$$

$$= k\frac{Q^2}{d^2}\left[.3535\hat{i} + 1.6565\hat{j}\right]$$